

FORM TP 2018299



TEST CODE 02134032

MAY/JUNE 2018

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 1 – Paper 032

ALGEBRA, GEOMETRY AND CALCULUS

1 hour 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of ONE question.
3. Answer ALL questions.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials Permitted

Mathematical formulae and tables (provided) – Revised 2012

Mathematical instruments

Silent, non-programmable, electronic calculator

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SECTION A

Module 1

Answer this question.

1. (a) Two propositions, **r** and **s** are given as

r: It is Monday

s: It is raining.

Express the following statements in symbolic form:

- (i) It is Monday and it is raining.

- (ii) It is not Monday or it is not raining.

[2 marks]

- (b) (i) Given that **p** and **q** are two propositions, complete the truth table below.

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim (p) \wedge \sim (q)$	$(p \vee q) \wedge [\sim (p) \wedge \sim (q)]$
T	T					
T	F					
F	T					
F	F					

[4 marks]

- (ii) Hence, state whether $(p \vee q) \wedge [\sim (p) \wedge \sim (q)]$ is a tautology or a contradiction. **Justify your response.**

[2 marks]

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- (c) Prove by mathematical induction that $2^{n+1} + 3^{2n-1}$ is a multiple of 7 for all positive integers $n \geq 1$.

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[7 marks]

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- (d) Solve the equation $\log_x (2x - 3) + \log_x (4) = 2$.

[5 marks]

Total 20 marks

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SECTION B

Module 2

Answer this question.

2. (a) A line, L_1 , has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and

a line, L_2 , has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ -4 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ where α and β are scalar parameters.

Given that L_1 and L_2 meet at a point A, determine

- (i) the coordinates of A

[3 marks]

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- (ii) the angle between line L_1 and L_2 , correct to 3 significant figures.

[3 marks]

- (iii) Show that the vector $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ is perpendicular to both L_1 and L_2 .

[3 marks]

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- (iv) Hence, determine the vector equation of the plane through the point $\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$, which is perpendicular to the vector $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

[3 marks]

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- (b) Solve the equation $\sin x + \cos 2x = 1$, where $0 \leq x \leq 2\pi$.

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[6 marks]

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- (c) Determine the Cartesian equation of the curve with parametric equations

$$x = 1 + t^2, y = 1 + t.$$

[2 marks]

Total 20 marks

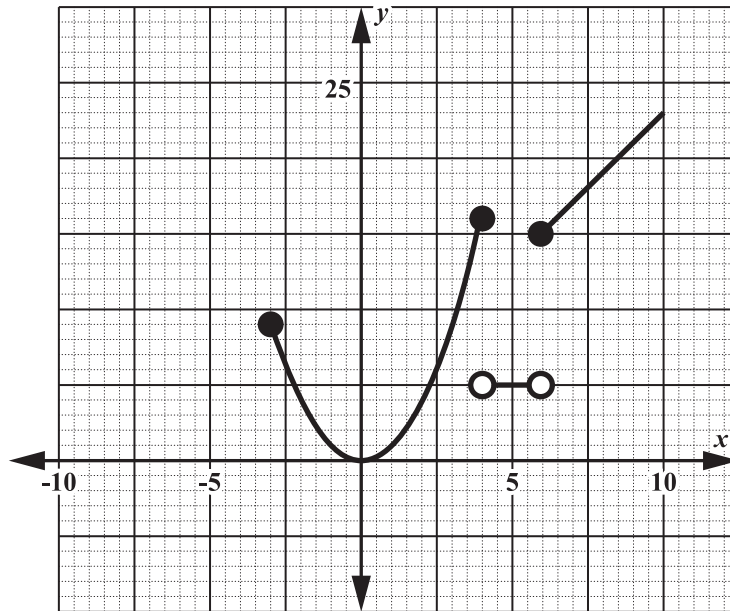
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SECTION C

Module 3

Answer this question.

3. (a) The diagram below shows the graph of $y = f(x)$.



- (i) Use the graph to find $f(6)$.

[1 mark]

- (ii) Determine whether the function is continuous at $x = 6$. **Justify your response.**

[2 marks]

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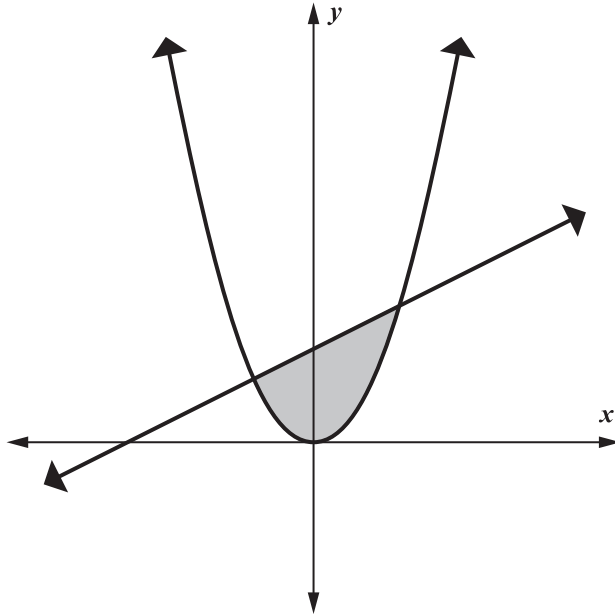
- (b) Using **first principles**, differentiate $f(x) = \sin 2x$.

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[6 marks]

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- (c) The diagram below (not drawn to scale) shows the curve $y = x^2$ and the line $y = x + 6$.



- (i) Determine the points of intersection of the curve $y = x^2$ and the line $y = x + 6$.

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[7 marks]

- (ii) Determine the area of the region bounded between the curve $y = x^2$ and the line $y = x + 6$.

[4 marks]

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.