

**FORM TP 2015269**



TEST CODE **02234032**

MAY/JUNE 2015

**CARIBBEAN EXAMINATIONS COUNCIL**

**CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®**

**PURE MATHEMATICS**

**UNIT 2 – Paper 032**

**ANALYSIS, MATRICES AND COMPLEX NUMBERS**

*1 hour 30 minutes*

**03 JUNE 2015 (a.m.)**

**READ THE FOLLOWING INSTRUCTIONS CAREFULLY.**

1. This examination paper consists of THREE sections.
2. Answer ALL questions from the THREE sections.
3. Each section consists of ONE question.
4. Write your solutions, with full working, in the answer booklet provided.
5. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.

**Examination Materials Permitted**

Graph paper (provided)

Mathematical formulae and tables (provided) – **Revised 2012**

Mathematical instruments

Silent, non-programmable, electronic calculator

**DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.**

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**SECTION A**

**Module 1**

**Answer this question.**

1. (a) A complex number  $z_1$  is such that  $|z_1| = 2$  and  $\arg z_1 = \frac{3\pi}{4}$ .
- (i) Identify the coordinates of  $z_1$  on an Argand diagram. **[3 marks]**
- (ii) On the same axes, connect  $z_1$  to the origin with a line segment and label the angle that represents  $\arg z_1$ . **[2 marks]**
- (iii) On the same axes, sketch the locus of the point  $z_2$  which moves in the complex plane such that  $|z_1 - z_2| = 1$ . **[2 marks]**
- (b) Use the trapezium rule with five ordinates to find an approximate value of
- $$\int_0^2 \sqrt{4 + x^3} \, dx. \quad \mathbf{[6 \text{ marks}]}$$
- (c) (i) Determine  $\int \frac{\sin^{-1}(\frac{x}{2})}{\sqrt{4 - x^2}} \, dx$ . **[5 marks]**
- (ii) Hence, calculate  $\int_0^1 \frac{\sin^{-1}(\frac{x}{2})}{\sqrt{4 - x^2}} \, dx$ . **[2 marks]**

**Total 20 marks**

**SECTION B**

**Module 2**

**Answer this question.**

2. (a) (i) Determine the Taylor series expansion of

$$f(x) = e^x \cos x$$

centred at  $\frac{\pi}{2}$  up to and including the first two non-zero terms. **[4 marks]**

- (ii) Hence, estimate  $f(\frac{\pi}{6})$ . **[2 marks]**

- (b) The twentieth term of an arithmetic progression is 35 and the sum of the first 19 terms is 285. Calculate the sum of the first five terms. **[7 marks]**

- (c) The numbers  $n - 4$ ,  $n + 2$ ,  $3n + 1$  are consecutive terms of a geometric sequence. Given that the corresponding series converges, determine the common ratio. **[7 marks]**

**Total 20 marks**

**SECTION C**

**Module 3**

**Answer this question.**

3. (a) A board game involves tossing TWO fair dice and ONE fair coin. The face shown on the coin determines the action of the next player.

If a HEAD is observed on the coin, the total on the dice is as observed. If a TAIL is observed on the coin, the number on each die must be 3 or less. If any of the numbers is more than 3, the die is thrown again until a 1, 2 or 3 is shown.

- (i) Copy and complete the table below to show the possible totals of the throws.

			DIE 1					
			1	2	3	4	5	6
HEAD	DIE 2	1	2	3	4	5		
		2	3	4	5	6		
		3	4	5	6	7		
		4	5	6	7	8		
		5						
		6						
TAIL	DIE 2	1						
		2						
		3						

**[2 marks]**

- (ii) What is the probability that the sum of the numbers on the dice is EVEN on any turn in the game? **[2 marks]**
- (iii) Determine the probability of obtaining a HEAD and an EVEN total on the dice. **[4 marks]**
- (iv) State, giving a reason for your answer, whether the events of obtaining a HEAD and an EVEN total on the dice are independent. **[2 marks]**

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(b) A differential equation is given as  $y' + y = 2 \sin x$ .

(i) Determine the general solution of the differential equation. **[8 marks]**

(ii) Hence, or otherwise, obtain the particular solution given that when  $x = 0, y = 1$ .  
**[2 marks]**

**Total 20 marks**

**END OF TEST**

**IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.**