TEST CODE **02134022**

FORM TP 2015317

MAY/JUNE 2015

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 1 – Paper 02

ALGEBRA, GEOMETRY AND CALCULUS

2 hours 30 minutes

16 JUNE 2015 (p.m.)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Answer ALL questions from the THREE sections.
3. Each section consists of TWO questions.
4. Write your solutions, with full working, in the answer booklet provided.
5. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A

Module 1

Answer BOTH questions.

1. (a) Let **p, q** and **r** be propositions.

(i) Copy and complete the truth table below for the propositions **(p ∧ q) → r** and **(p → r)** ∧ **(q → r)**.

p	q	r	p ∧ q	p → r	q → r	(p ∧ q) → r	(p → r) ∧ (q → r)
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

[5 marks]

(ii) Hence, determine whether or not **(p ∧ q) → r** and **(p → r) ∧ (q → r)** are logically equivalent. **Justify your response.** **[2 marks]**

(b) Use mathematical induction to prove that $10^{n+1} + 3(10^n) + 5$ is divisible by 9 for all natural numbers. **[8 marks]**

(c) Solve the equation $x^3 - 6x^2 - 69x + 154 = 0$. **[10 marks]**

Total 25 marks

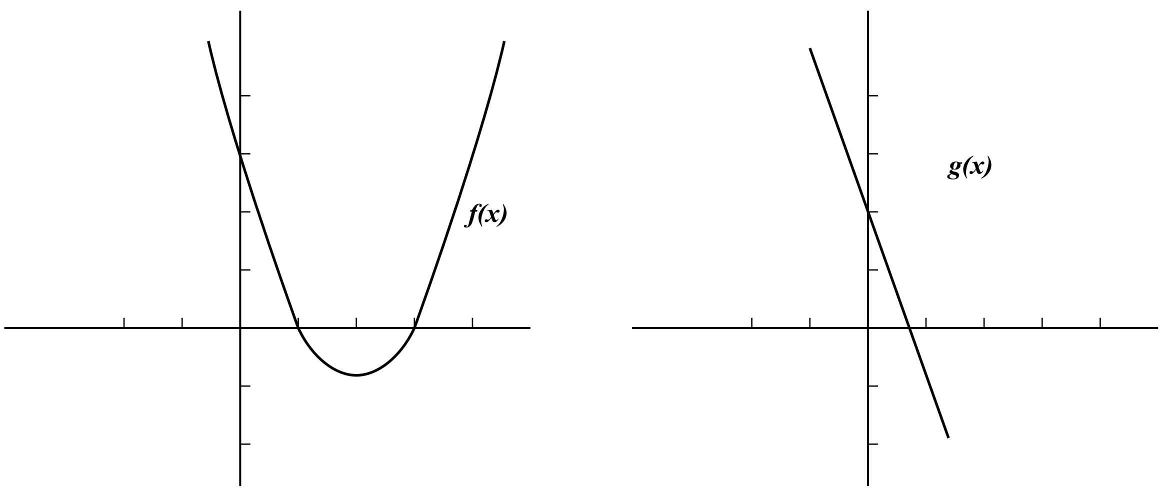
2. (a) (i) Show that $\frac{1}{\log_a x}$ may be rewritten as $\frac{\ln a}{\ln x}$, where a > 0, a ≠ 1.
[5 marks]

(ii) Hence, or otherwise, solve the equation:

$$\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + \frac{1}{\log_4 x} + \frac{1}{\log_5 x} = 1$$

[4 marks]

(b) The diagrams below (**not drawn to scale**) show the graphs of $f(x) = x^2 - 4x + 3$ and $g(x) = 2 - 3x$.



(i) On the **same axes**, sketch the graphs $|f(x)|$ and $|g(x)|$. **[4 marks]**

(ii) Hence, or otherwise, solve the equation $|x^2 - 4x + 3| = |2 - 3x|$. **[6 marks]**

(c) A function f is such that $f^{-1}(x) = \sqrt{x + 1} + 1$.

Determine the value of $f(f^{-1}[f(1)])$. **[6 marks]**

Total 25 marks

SECTION B

Module 2

Answer BOTH questions.

3. (a) Given that $\tan^2 x = \dfrac{\sin^2 x}{1 - \sin^2 x}$, show that $\sin x = \pm \dfrac{\tan x}{\sqrt{1 + \tan^2 x}}$. **[6 marks]**

Note: You are not required to simplify the expression $\sin^2 x$.

(b) (i) Show that $f(\theta) = \sqrt{3}\sin\theta + \cos\theta$ may be expressed as $f(\theta) = 2\sin\left(\theta + \dfrac{\pi}{6}\right)$ where $0 \le \theta \le \dfrac{\pi}{2}$. **[6 marks]**

(ii) Hence, or otherwise,

a) solve the equation $f(\theta) = \sqrt{2}$ for $0 \le \theta \le 2\pi$ **[5 marks]**

b) determine the maximum value of f and the **smallest** positive value of θ for which it occurs. **[3 marks]**

(c) Without the use of a calculator or tables, find the exact value of $\cos \dfrac{\pi}{12}$. **[5 marks]**

Total 25 marks

4. (a) Calculate the distance between the points of intersection of the line $3x - 2y + 6 = 0$ and the circle $x^2 + y^2 = 9$. **[8 marks]**

(b) A circle, C, is defined by the equation $3x^2 + 3y^2 - 2y - 4x = 0$.

(i) Find the centre and radius of C. **[5 marks]**

(ii) Find the equation of the tangent to C at the point (0,0) . **[4 marks]**

(c) Let $\mathbf{a} = c\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

Find c given that the angle between the vectors **a** and **b** is $\dfrac{\pi}{3}$. **[8 marks]**

Total 25 marks

SECTION C

Module 3

Answer BOTH questions.

5. (a) Given that $y = \tan^{-1}(1 - x^2)$, find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. **[5 marks]**

(b) The equation of a circle is given by the parametric equations

$$x = a + b \sin t \text{ and } y = c + d \cos t \text{ where } b \neq 0.$$

Determine

(i) $\dfrac{dy}{dx}$ in terms of t **[4 marks]**

(ii) the value of t for which the circle has vertical tangents. **[3 marks]**

(c) A function f is defined as $f(x) = 3 + \dfrac{6}{x-2}$.

(i) Determine

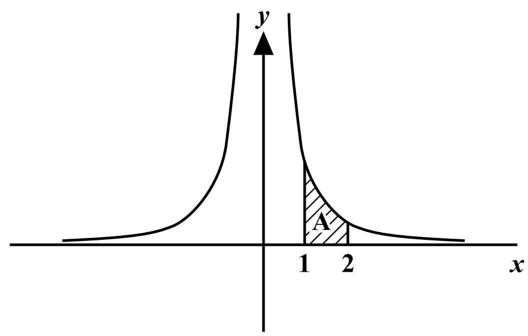
a) whether the function f has turning points **[4 marks]**

b) the vertical and horizontal asymptotes of f. **[4 marks]**

(ii) Hence, sketch the graph of f. **[5 marks]**

Total 25 marks

6. (a) The diagram below **(not drawn to scale)** shows the curve $f(x) = \frac{1}{x^2}$.



(i) Determine the area of the region, A. **[3 marks]**

(ii) Calculate the volume of the solid that results from revolving the region, A, about the x-axis. **[5 marks]**

(b) The gradient function for a curve $y = f(x)$ is given as $\frac{dy}{dx} = -\frac{y}{x}$.

Given that the curve passes through the point (1,1), determine an expression for $f(x)$.

[7 marks]

(c) By using the substitution, $y = x^2$, show that $\int \frac{1}{\sqrt{y} + \sqrt{y^3}}\, dy = \int \frac{2}{1 + x^2}\, dx$. **[5 marks]**

(d) Given that $\sin 2\theta = 2 \sin\theta \cos\theta$ and $\cos 2\theta = 1 - 2 \sin^2 \theta$,
find $\int \sin^2 x \cos^2 x \, dx$.

[5 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.