FORM TP 2012232

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – Paper 032

ALGEBRA, GEOMETRY AND CALCULUS

1 hour 30 minutes

08 JUNE 2012 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.
The maximum mark for each Module is 20.
The maximum mark for this examination is 60.
This examination consists of 4 printed pages.

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A (Module 1)

Answer this question.

1. (a) The roots of the cubic equation $x^3 - px - 48 = 0$ are α, 2α and -3α.

Find

(i) the value of α **[3 marks]**

(ii) the value of the constant p. **[4 marks]**

(b) Prove by mathematical induction that

$9^n - 1$ is divisible by 8 for all integers $n \geq 1$. **[6 marks]**

(c) Let m and n be positive integers.

(i) Prove that $\log_n m = \dfrac{1}{\log_m n}$. **[3 marks]**

(ii) Hence, solve for x, the equation

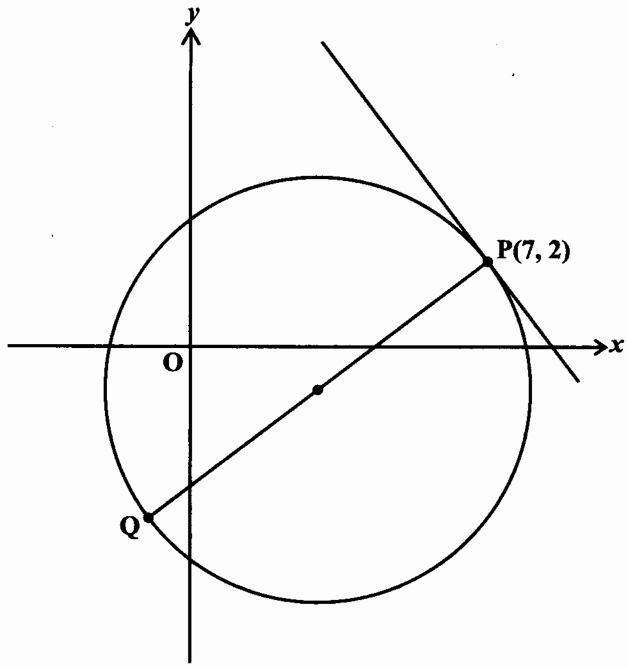
$\log_2 x + 2\log_x 2 = 3$. **[4 marks]**

Total 20 marks

SECTION B (Module 2)

Answer this question.

2. (a) The diagram below **(not drawn to scale)** shows the graph of the circle, **C**, whose equation is $x^2 + y^2 - 6x + 2y - 15 = 0$.



(i) Determine the radius and the coordinates of the centre of **C**. **[3 marks]**

(ii) Find the equation of the tangent to the circle at the point P (7, 2). **[5 marks]**

(iii) Find the coordinates of the point Q ($Q \neq P$) at which the diameter through P cuts the circle. **[2 marks]**

(b) (i) Express $f(\theta) = 3\sqrt{3}\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$ where $R > 0$ and θ is acute.

[4 marks]

(ii) Hence, obtain the maximum value of $f(\theta)$. **[2 marks]**

(c) The vector $\overrightarrow{PQ} = \mathbf{i} - 3\mathbf{j}$ is parallel to the vector $\overrightarrow{OR}$ with $|\overrightarrow{OR}| = \sqrt{5}$. Find scalars a and b such that $\overrightarrow{OR} = a\mathbf{i} + b\mathbf{j}$. **[4 marks]**

Total 20 marks

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SECTION C (Module 3)

Answer this question.

3. (a) (i) By expressing $x - 4$ as $(\sqrt{x} + 2)(\sqrt{x} - 2)$, find

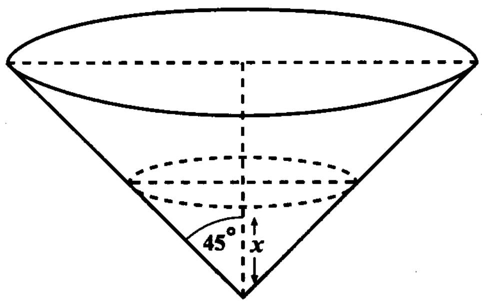
$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}.$$ **[3 marks]**

(ii) Hence, or otherwise, find

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x^2 - 5x + 4}.$$ **[5 marks]**

(b) Find the gradient of the curve $y = 2x^3$ at the point P on the curve at which $y = 16$. **[3 marks]**

(c) The diagram below **(not drawn to scale)** represents an empty vessel in the shape of a right circular cone of semi-vertical angle 45°. Water is poured into the vessel at the rate of 10 cm^3 per second. At time, t, seconds after the start of the pouring of water, the height of the water in the vessel is x cm and its volume is V cm^3.



(i) Express V in terms of t only. **[1 mark]**

(ii) Express V in terms of x only. **[2 marks]**

(iii) Find, **correct to 2 decimal places**, the rate at which the water level is rising after 5 seconds. **[6 marks]**

Total 20 marks

[The volume of a right circular cone of height h and radius of base r is $V = \frac{1}{3}\pi r^2 h$.]

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.