

FORM TP 2009235



TEST CODE **02134032**

MAY/JUNE 2009

**CARIBBEAN EXAMINATIONS COUNCIL
ADVANCED PROFICIENCY EXAMINATION**

PURE MATHEMATICS

UNIT 1 – PAPER 03/B

ALGEBRA, GEOMETRY AND CALCULUS

1 ½ hours

10 JUNE 2009 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.

The maximum mark for each Module is 20.

The maximum mark for this examination is 60.

This examination consists of 3 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)

Mathematical formulae and tables (provided) – **Revised 2009**

Mathematical instruments

Silent, non-programmable, electronic calculator

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SECTION A (Module 1)

Answer this question.

1. (a) Find the set of real values of x for which

$$|x - 1| > |2x + 1|. \quad \text{[6 marks]}$$

- (b) A packaging company makes crates for special purposes. The company finds that the unit cost $f(x)$, in thousands of dollars, of producing crates with a square base of x metres is

$$f(x) = (x^2 - 4x)^2 + 2x^2 - 8x.$$

Using the substitution $y = x^2 - 4x$, find the sizes of the crates for which the unit cost is three thousand dollars. [7 marks]

- (c) (i) By taking logarithms, show that for any positive integers p and x ,

$$p^{\log_p x} = x. \quad \text{[4 marks]}$$

- (ii) Hence, **without using calculators or tables**, find the EXACT value of

$$2^{(\log_2 6 + \log_2 15 - 2 \log_2 3)}. \quad \text{[3 marks]}$$

Total 20 marks

SECTION B (Module 2)

Answer this question.

2. (a) (i) Show that the equation of the tangent to the circle

$$x^2 + y^2 + 8x + 14 = 0 \text{ at the point } (p, q) \text{ is}$$

$$(p + 4)(x - p) + q(y - q) = 0. \quad \text{[3 marks]}$$

- (ii) Show that the equation of the tangent can also be written as

$$px + qy + 4(x + p) + 14 = 0. \quad \text{[2 marks]}$$

- (iii) If the tangent at (p, q) on the circle passes through the point $(-3, 3)$, find the values of p and q . [7 marks]

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- (b) A point moves so that at time t its distances from the coordinate axes are given by

$$x = 2 + 3 \cos t \text{ and } y = 4 + 4 \sin t.$$

- (i) Find the maximum and minimum values of x and y . [4 marks]
- (ii) Find the Cartesian equation of the curve traced by the point. [4 marks]

Total 20 marks

SECTION C (Module 3)

Answer this question.

3. (a) Find $\int \left(\frac{t^6 + 3t - 1}{t^5} \right) dt$. [5 marks]

- (b) The point $P(-1, 5)$ is a point of inflexion on the curve $y = x^3 + bx^2 + c$, where b and c are constants.

Find

- (i) the values of b and c [5 marks]
- (ii) the equation of the normal to the curve at P . [3 marks]
- (c) Scientists on an experimental station released a spherical balloon into the atmosphere. The volume of air in the balloon is increased or decreased as required.
- (i) The radius, r , of the balloon is increasing at the constant rate of 0.02 cm/s. Find the rate at which the volume, $V \text{ cm}^3$, is increasing when $r = 3$ cm. Express your answer in terms of π . [2 marks]
- (ii) The volume, $V \text{ cm}^3$, of the balloon decreases by 6% when the radius decreases by $p\%$. Find p . [5 marks]

Total 20 marks

END OF TEST