Mathematics for Business

MAT106

May 2007

SECTION 1

Instructions: Answer ALL questions in this section. Write your answer together with any working on your answer script.

1. Solve simultaneously: $2x - y = 6 \;\; \& \;\; 4x + 3y = 22$

2. Write the next two terms in the sequence $x, -2x^2, 4x^3, \ldots$

3. How many numbers are there between 26 and 190 that are divisible by 7 ?

4. Find the remainder when $x^3 + 4x^2 - 3x + 2$ is divided by $(x + 5)$

5. Simplify $\left(\frac{8}{27}\right)^{\frac{2}{3}}$

6. Evaluate $\int_4^9 x^{-\frac{1}{2}} dx$

7. Factorise $5x^2 - 13x - 6$

8. Find the gradient of the tangent line to the curve $y = x^3 - 1$ at the point $(1, 0)$

9. Find the sum to ten terms of the progression $-1, 3, 7, 11, \ldots$

10. Find $\frac{d^2y}{dx^2}$ if $y = 3x^2 - \frac{1}{x}$

11. Simplify $2\log_{10} 20 - (\log_{10} 5 + \log_{10} 8)$

[Total 11x2 = 22]

SECTION 2

Instructions: There are nine (9) questions in this section.
Answer QUESTION ONE & FIVE OTHERS.

1. A diet is to contain at least 16 units of Carbohydrates and 20 units of Protein and 12 units of Fats. One package of Food A contains 2 units of Carbohydrates and 4 units of Protein and 1 unit of Fats. One package of Food B contains 2 units of Carbohydrates and 1 unit of Protein and 3 units of Fats. Food A costs $1.50 per pkg and Feed B costs $1.00 per pkg. Use Linear Programming to identify how many packages of each food should be purchased in order to minimise cost. Calculate the minimum cost.

[18]

2. Express in partial fractions:

   (a) $\dfrac{2x^2+1}{x\,(x-1)^2}$

   [5]

   (b) $\dfrac{x^3+2x+1}{x\,(x^2+1)}$

   [7]

3. (a) Given that $(2x+1)$ is a factor of $p\,(x) = 2x^3 + ax^2 + 16x + 6$, show that $a = 9$.

   [3]

   (b) Hence, or otherwise, find the real quadratic factor of $p\,(x)$.

   [4]

   (c) By completing the square, show that the quadratic factor of $p\,(x)$ is positive for all real values of x.

   [4]

   (d) State the number of real roots of $p\,(x)$.

   [1]

4. (a) Solve the following system of equations:

$$\begin{aligned} x + 2y + 3z &= 11 \\ 2y - 4z &= -6 \\ -x + y + 2z &= 2 \end{aligned}$$

[8]

(b) At a certain factory, the daily output is given by $Q(L) = 20,000L^{\frac{1}{2}}$ units where L denotes the size of the labour force measured in worker-hours.

Find the marginal output when $L = 900$ worker-hours.

Is the output increasing when $L = 900$ worker-hours? Explain.

[4]

5. (a) The fifth term of an arithmetic series is 14 and the sum of the first three terms of the series is -3.

   i. Use algebra to show that the first term of the series is -6 and calculate the common difference of the series.

   ii. Given that the nth term of the series is greater than 282, find the least possible value of n.

[7]

(b) Solve the simultaneous equations:

$$\begin{aligned} x - 3y &= 6 \\ 3xy + x &= 24 \end{aligned}$$

[5]

6. A firm has determined that market demand for its goods behaves according to $p = -\frac{1}{2}q^2 + 60$, where p is the price per unit and q is the quantity demanded. Further, it is known that the fixed costs are 35 and that the variable costs can be shown by $VC = 20q + 5q^2 - 0.5q^3$

(a) Derive formulas for:

   i. Revenue

   ii. Total Costs

   iii. Profit

[6]

(b) Find the value of q that maximises profit and hence find the maximum profit.

[4]

(c) Prove that this value does not give the maximum for revenue.

[2]

7. (a) The sum to infinity of a geometric series is three times the sum to 2 terms. Find all possible values of the common ratio.

[4]

(b) Find the derivative, but do not simplify:

i. $(3x^2 + 5x + 2)(7x + 5)$

ii. $\dfrac{x^2 - 1}{\sqrt{x + 3}}$

iii. $\left(x + \frac{2}{x}\right)^{-\frac{3}{2}}$

[8]

8. (a) Carol starts a new job on a salary of \$20000. She is given an annual wage rise of \$500 at the end of every year until she reaches her maximum salary of \$25000. Find the total amount she earns (assuming no other rises),

i. in the first 10 years and

ii. over 15 years.

[5]

(b) At the beginning of the year 2000 a company bought a new machine for \$15 000. Each year the value of the machine decreases by 20% of its value at the start of the year.

i. Show that at the start of the year 2002, the value of the machine was \$9600.

ii. When the value of the machine falls below \$500, the company will replace it. Find the year in which the machine will be replaced.

iii. To plan for a replacement machine, the company pays \$1000 at the start of each year into a savings account. The account pays interest of 5% per annum. The first payment was made when the machine was first bought and the last payment will be made at the start of the year in which the machine is replaced. Using your answer to part (ii), find how much the savings account will be worth when the machine is replaced.

[7]

9. (a) Without the use of graph paper, sketch the graph of $y = -2x^2 + x + 3$ showing all the intercepts and the turning point.

[9]

(b) Use the quadratic formula to solve $3x^2 = -8x + 5$

[3]

[1] Compiled by C.Ormerod (admin@etoipi.com)